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DIRICHLET AVERAGE OF ADVANCED MODIFIED M- FUNCTION AND FRACTIONAL DERIVATIVE

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ABSTRACT

In this work we set up a relation between Dirichlet average of Advanced Modified M- function [16] and fractional derivative.

KEYWORDS AND PHRASES: Dirichlet average Advanced Modified M- function,, fractional derivative and Fractional calculus operators.

]**Mathematics Subject Classification**: 26A33, 33A30, 33A25 and 83C99.

INTRODUCTION

Carlson [1-5] has defined Dirichlet average of functions which represents certain type of integral average with respect to Dirichlet measure. Carlson[1-5] showed that various important special functions can be derived as Dirichlet averages for the ordinary simple functions likex^t,e^x etc. He has also pointed out that the hidden symmetry of all special functions which provided their various transformations can be obtained by averaging x^n,e^x etc. Thus he established a unique process towards the unification of special functions by averaging a limited number of ordinary functions. Almost all known analytic special functions and their well known properties have been derived by this process.

In this paper, the Dirichlet average of Sharma's **Advanced Modified M − function [16]**, has been obtained.

DEFINITIONS

We give blew some of the definitions which are necessary in the preparation of this paper.

Standard Simplex in $R^n, n \geq 1$:

We denote the standard simplex in R^n, $n \geq 1$ by [1, p.62].

$$E = E_n = \{S(u_1, u_2, \dots \dots . . u_n) \ : \ u_1 \geq 0, \dots \dots \dots . u_n \geq 0, \ u_1 + u_2 + \cdots \dots \dots + u_n \leq 1\} \qquad (2.1.1)$$

Dirichlet measure:

Let $b \in C^k, k \geq 2$ and let $E = E_{k-1}$ be the standard simplex in R^{k-1}. The complex measure μ_b is defined by E[1].

$$d\mu_b(u) = \frac{1}{B(b)} u_1^{b_1-1} \dots \dots \dots \dots . u_{k-1}^{b_{k-1}-1} (1 - u_1 - \cdots \dots \dots - u_{k-1}) b_k^{-1} du_1 \dots \dots \dots . du_{k-1} \qquad (2.2.1)$$

Will be called a Dirichlet measure.

Here

$$B(b) = B(b1, \dots \dots . bk) = \frac{\Gamma(b_1) \dots \dots \dots \dots . . \Gamma(b_k)}{\Gamma(b_1 + \cdots \dots \dots . . + b_k)},$$

$$C_> = \{z \in z : z \neq 0, |ph\, z| < \pi/2\},$$

Open right half plane and $C_>$k is the k^{th} Cartesian power of $C_>$

Dirichlet Average[1, p.75]:

Let Ω be the convex set in $C_>$, let $z = (z_1, \dots \dots \dots , z_k) \in \Omega^k, k \geq 2$ and let $u.z$ be a convex combination of $z_1, \dots \dots \dots , z_k$. Let f be a measureable function on Ω and let μ_b be a Dirichlet measure on the standard simplex E in R^{k-1}.Define

$$F(b,z)=\int_E f(u.z)d\,\mu_b(u) \qquad (2.3.1)$$

We shall call F the Dirichlet measure of f with variables $z=(z_1,\dots\dots\dots,z_k)$ and parameters $b=(b_1,\dots\dots\dots.b_k)$.
Here

$$u.z=\sum_{i=1}^{k}u_i z_i \text{ and } u_k=1-u_1-\cdots\dots\dots.-u_{k-1} \qquad (2.3.2)$$

If $k=1$, define $F(b,z)=f(z)$.

Fractional Derivative [8, p.181]:
The concept of fractional derivative with respect to an arbitrary function has been used by Erdelyi[8]. The most common definition for the fractional derivative of order α found in the literature on the "Riemann-Liouville integral" is

$$D_z^{\alpha}F(z)=\frac{1}{\Gamma(-\alpha)}\int_0^z F(t)(z-t)^{-\alpha-1}dt \qquad (2.4.1)$$

Where $Re(\alpha)<0$ and $F(x)$ is the form of $x^p f(x)$, where $f(x)$ is analytic at $x=0$.

Advanced Modified M – Function –

We give the new special function, called **Advanced Modified M – function [16]**,, which is the most generalization of New Generalized Mittag-Leffler Function . Here, we give first the notation and the definition of the New Special **Advanced Modified M – function**, introduced by the author as follows:

$${}^{\alpha,\beta,\gamma,\delta,\rho}_{p}\mathbf{M}_q^{k_1,\dots k_p,l_1,\dots l_q;c}(t)=\sum_{n=o}^{\infty}\frac{(a_1)_n\ \dots\ (a_p)_n\,(\gamma)_n}{(b_1)_n\dots(b_q)_n}\frac{(\delta)_n}{(\rho)_n}\frac{k_1^n\dots k_p^n}{l_1^n\dots l_q^n}\frac{\prod_{i=1}^{n}a_i^{a_i}}{\prod_{i=1}^{n}b_i^{b_i}}\frac{(ct)^{(n+\gamma)\alpha-\beta-1}}{n!\,\Gamma\big((n+\gamma)\alpha-\beta\big)} \qquad (3.1)$$

There are p upper parameters $a_1,a_2,\dots a_p$ and q lower parameters $b_1,\ b_2,\dots b_q,\alpha,\ \beta,\ \gamma,\delta,\rho\ \epsilon\ C, Re(\alpha)>0,\ Re(\beta)>0,\ Re(\gamma)>0,\ Re(\delta)>0,\ Re(\rho)>0,\ Re(\alpha\gamma-\beta)>0$and $(a_j)_k$ $(b_j)_k$ are pochammer symbols and $k_1,\dots k_p$, $l_1,\dots l_q$are constants. The function (1) is defined when none of the denominator parameters $b_j s,\ j=1,2,\dots q$ is a negative integer or zero. If any parameter a_j is negative then the function (1) terminates into a polynomial in (t).

EQUIVALENCE

In this section , we shall show the equivalence of single Dirichlet average of **Advanced Modified M – function**, $(k=2)$ with the fractional derivative i.e.

$$S(\beta,\beta';x,y)=\frac{\Gamma(\beta+\beta')}{\Gamma\beta}(x-y)^{1-\beta-\beta'}D_{x-y}^{-\beta'}\,{}^{\alpha,\beta,\gamma,\delta,\rho}_{p}\mathbf{M}_q^{k_1,\dots k_p,l_1,\dots l_q;c}(x)(x-y)^{\beta-1} \qquad (4.1)$$

Proof:

$$S(\beta,\beta';x,y)={}^{\alpha,\beta,\gamma,\delta,\rho}_{p}\mathbf{M}_q^{k_1,\dots k_p,l_1,\dots l_q;c}(t)$$

$$=\sum_{n=o}^{\infty}\frac{(a_1)_n\ \dots\ (a_p)_n\,(\gamma)_n}{(b_1)_n\dots(b_q)_n}\frac{(\delta)_n}{(\rho)_n}\frac{k_1^n\dots k_p^n}{l_1^n\dots l_q^n}\frac{\prod_{i=1}^{n}a_i^{a_i}}{\prod_{i=1}^{n}b_i^{b_i}}\frac{(ct)^{(n+\gamma)\alpha-\beta-1}}{n!\,\Gamma\big((n+\gamma)\alpha-\beta\big)}\ R_{(n+\gamma)\alpha-\beta-1}(\beta,\beta';x,y)$$

$$=\sum_{n=o}^{\infty}\frac{(a_1)_n\ \dots\ (a_p)_n\,(\gamma)_n}{(b_1)_n\dots(b_q)_n}\frac{(\delta)_n}{(\rho)_n}\frac{k_1^n\dots k_p^n}{l_1^n\dots l_q^n}\frac{\prod_{i=1}^{n}a_i^{a_i}}{\prod_{i=1}^{n}b_i^{b_i}}\frac{(c)^{(n+\gamma)\alpha-\beta-1}}{n!\,\Gamma\big((n+\gamma)\alpha-\beta\big)}\frac{\Gamma(\beta+\beta')}{\Gamma\beta\,\Gamma\beta'}\int_0^1[ux+(1-u)y]^{((n+\gamma)\alpha-\beta-1)}\,u^{\beta-1}(1-u)^{\beta'-1}du$$

Putting $u(x-y)=t$, we have,

$$=\sum_{n=o}^{\infty}\frac{(a_1)_n\ \dots\ (a_p)_n\,(\gamma)_n}{(b_1)_n\dots(b_q)_n}\frac{(\delta)_n}{(\rho)_n}\frac{k_1^n\dots k_p^n}{l_1^n\dots l_q^n}\frac{\prod_{i=1}^{n}a_i^{a_i}}{\prod_{i=1}^{n}b_i^{b_i}}\frac{(c)^{(n+\gamma)\alpha-\beta-1}}{n!\,\Gamma\big((n+\gamma)\alpha-\beta\big)}\frac{\Gamma(\beta+\beta')}{\Gamma\beta\,\Gamma\beta'}\int_0^{x-y}[t+y]^{((n+\gamma)\alpha-\beta-1)}\left(\frac{t}{x-y}\right)^{\beta-1}\left(1-\frac{t}{x-y}\right)^{\beta'-1}\frac{dt}{x-y}$$

On changing the order of integration and summation, we have

$$= (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\,\Gamma\beta'} \int_0^{x-y} \sum_{n=o}^{\infty} \frac{(a_1)_n \ \dots \ (a_p)_n (\gamma)_n}{(b_1)_n \dots (b_q)_n} \frac{(\delta)_n}{(\rho)_n} \frac{k_1^n \dots k_p^n}{l_1^n \dots l_q^n} \frac{\prod_{i=1}^n a_i{}^{a_i}}{\prod_{i=1}^n b_i{}^{b_i}} \frac{(c)^{(n+\gamma)\alpha-\beta-1}}{n!\,\Gamma\big((n+\gamma)\alpha-\beta\big)} [t + y]^{((n+\gamma)\alpha-\beta-1)}\,(t)^{\beta-1}(x-y-t)^{\beta'-1}dt$$

Or

$$= (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta\,\Gamma\beta'} \int_0^{x-y} {}^{\alpha,\beta,\gamma,\delta,\rho}_{\;\;p}\mathbf{M}_q^{k_1,\dots k_p,l_1,\dots l_q;c}(x)\,(t)^{\beta-1}(x-y-t)^{\beta'-1}dt$$

Hence by the definition of fractional derivative, we get

$$S(\beta,\beta';x,y) = (x-y)^{1-\beta-\beta'} \frac{\Gamma(\beta+\beta')}{\Gamma\beta} D_{x-y}^{-\beta'}\, {}^{\alpha,\beta,\gamma,\delta,\rho}_{\;\;p}\mathbf{M}_q^{k_1,\dots k_p,l_1,\dots l_q;c}(x)(x-y)^{\beta-1}$$

This completes the Analysis.

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